Commentary

We are submitting the following note as a technical criticism of a recent paper in your journal, Color Vision and the Four-Color-Map Problem, by Purves et al. (JOCN 12:2, 233–237, 2000), which asserts that the need for four opponent colors in Hering's theory are "explained by the four-color theorem of mathematics."

Unfortunately, there is a basic misunderstanding of the four-color theorem as interpreted by Purves et al. This theorem requires four discrete labels, not the four parameters, three dimensions of Hering's theory, to label a planar graph. In fact, a single dimension, e.g., the rod channel, is more than sufficient to provide many dozens of gray-scale labels, even in the absence of the color system entirely. The four-parameter theory of Hering provides many thousands of discriminable "labels" if one wishes to interpret segmenting the visual scene as a combinatorial labeling problem.

The most that can be said in this context is that the four-color theorem provides a lower limit for the number of discriminable color "labels" required by a color visual system. Humans have four parameters of color vision (according to Hering's theory, at least), but we can discriminate (let us arbitrarily and conservatively say) 1,000 different colors.

The four-color theorem then (using the arbitrary number 1,000 colors) allows us to state that

$$1,000 > 4 \tag{1}$$

This is a weak enough lower bound to provide no constraint at all on possible models of color vision.

Sincerely,

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Reply

The question raised by Schwartz and Cohen—why any four-color percepts cannot suffice to solve the four-color-map problem in vision—is specifically addressed in the section of our paper entitled "Why not any four colors?" (p. 236). To remind readers, the four-color-map problem refers to the challenge of proving that any arbitrarily complex map requires only four colors to distinguish its territories. (Although the problem was posed in the nineteenth century, it was not solved until the 1970s.) The purpose of our paper was to suggest that this logical problem in topology is pertinent to vision. With respect to the question raised by Cohen and Schwartz, the key points we made are: (1) the four-color-map problem can indeed be solved in principle by any four qualities that are distinct; (2) however, in solving this problem in vision, the visual system, unlike the cartographer, must preserve the full range of perceptual relationships between the spectral returns from real-world objects (since distinguishing surfaces by virtue of these relationships is the purpose of color vision); (3) consequently, four *dimensions* (or categories) of color sensation are needed, not just four colors; and (4) the four distinct dimensions of human color vision we experience (red, green, blue, and yellow, each defined by a unique hue) appear to achieve a biological solution of the four-color-map problem, and may have evolved in whole or in part for this purpose. While it is obviously true that the visual system generates "many thousands of discriminable labels," to experience both the full range of color percepts *and* to solve this topological problem, a comparison of color sensations in four different directions is needed.

The importance of this topological constraint in optimally distinguishing surfaces based on their spectral returns provides a plausible reason for the four dimensions of hue that we experience. Indeed, we know of no other rationale for this four-dimensional quality of color vision, which, to the extent that it has been considered at all, is assumed to be an incidental consequence of color opponency. We present this suggestion about the nature of color sensation as an example of how otherwise-mysterious aspects of visual perception can be understood in terms of the demands made by the characteristics of the world we live in, in this case, the topological requirement of four different qualities to unambiguously distinguish adjacent surfaces in any two-dimensional topology.

Yours sincerely,

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