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# REGENERATION AMPLITUDE OF $K_S^0$ FROM $K_L^0$ IN CARBON *

W.C. CARITHERS **, T. MODIS ***, D.R. NYGREN **, T.P. PUN ‡,
E.L. SCHWARTZ ‡‡ and H. STICKER ‡‡‡
*Columbia University, New York, NY 10027*

J.H. CHRISTENSON
*New York University, New York, NY*

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We have measured the transmission regeneration amplitude for carbon. The invariant amplitude $[f(0) - \bar{f}(0)]/P$ varies from 5.909 ± 0.066 mb at 4.5 GeV/$c$ to 3.933 ± 0.152 mb at 9.5 GeV/$c$. The results agree with optical model calculations using measured kaon-nucleon total cross sections. The data also provide a value for the $\omega$ trajectory intercept $\alpha_\omega(0) = 0.42 \pm 0.04$.

A $K_L^0$ beam traversing a block of material causes a $K_S^0$ component to be regenerated since the $K^0$ and $\bar{K}^0$ components interact differently with matter. In the region following the regenerator, the beam is a superposition of the short- and long-lived states: $|\psi\rangle \to |K_L\rangle + \rho|K_S\rangle$. The coherent regeneration amplitude $\rho$ is defined as the ratio of the short-lived to the long-lived amplitudes in the forward direction after traversal and is given by [1]

$$\rho = \pi N L i \frac{[f(0) - \bar{f}(0)]}{P} \frac{1 - \exp[i\Delta m - \frac{1}{2}\Gamma_S \, mL/P]}{-(i\Delta m - \frac{1}{2}\Gamma_S \, mL/P)}, \tag{1}$$

where $N$ = the density of the scatterers; $m = \frac{1}{2}(m_L + m_S)$ the average kaon mass; $P$ = beam momentum; $\Delta m = m_L - m_S$ is the $K_L$, $K_S$ mass difference; $L$ = the thickness of the regenerator; $f(0)$, $\bar{f}(0)$ = the $K^0$ and $\bar{K}^0$ elastic forward scattering amplitudes. Clearly, knowledge of $m$, $\Delta m$, and $\Gamma_S$ combined with a measurement of $\rho$

* Research supported by the National Science Foundation.
** Present address: Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720.
*** Present address: CERN, Geneva, Switzerland.
‡ Present address: SLAC, Stanford, CA 94305.
‡‡ Present address: Brain Research Laboratory, New York Medical College, New York, NY.
‡‡‡ Present address: Rockefeller University, New York, NY 10021.

determines $f(0) - \bar{f}(0)$. We determine $\Gamma_S$ from our own data and express the result for $|f(0) - \bar{f}(0)|$ as a function of $\Gamma_S$.

$K \to \pi^+\pi^-$ decays were detected with a spectrometer employing three X–Y multiwire proportional chambers. The details of the apparatus have been described in previous publications [2,3]. The spectrometer magnet was operated at 131.6 MeV/$c$ transverse momentum to optimize acceptance for $2\pi$ and leptonic decays. Electrons were detected in an atmospheric pressure hydrogen gas Čerenkov counter with 12 independent optical sectors. Muons were identified by their ability to penetrate 900 g/cm$^2$ of concrete and register in a 16 element scintillation counter hodoscope. The normal trigger required exactly two hits in each of the two planes (horizontal and vertical) of the three chambers. $\pi^+\pi^-$ decay candidates are further required to meet the following criteria: (i) inbending events only; (ii) no electron or muon signature; (iii) a $\chi^2 \leqslant 16$ for three degrees of freedom (the vertical kinks of each track and the closest distance of the approach of the two reconstructed trajectories in space); (iv) the $\pi^+\pi^-$ invariant mass to be within two standard deviations of the kaon mass; (v) the momentum $P_i$ of individual tracks to be $1.4 < P_i < 7$; (vi) the kaon momentum $P_K$ to be $4 < P_K < 10$ GeV/$c$.

These cuts removed all but a negligible number of $\Lambda^0 \to p\pi$ events that could simulate $K \to \pi^+\pi^-$. A small background of $K_{\mu 3}$ decays was subtracted by extrapolating under the $2\pi$ peak in the invariant mass distribution. The remaining events consisted of pure coherent and incoherent $\pi\pi$ decays. The incoherent events were removed for each proper time and each momentum bin by extrapolating the transverse momentum squared distribution, $P_\perp^2 > 0$ to $P_\perp^2 = 0$ and subtracting from the measured intensity. Because carbon posesses a large diffraction angle the subtraction never exceeded a few percent.

The coherent $\pi^+\pi^-$ intensity is expected to be of the form:

$$I_{+-}(p,\tau) = \alpha\epsilon(P,\tau)\left\{\left|\frac{\rho}{\eta_{+-}}\right|^2 e^{-\Gamma_S\tau} + e^{-\Gamma_L\tau} + 2\left|\frac{\rho}{\eta_{+-}}\right| e^{-\Gamma\tau}\cos(\Delta m\tau + \phi_T)\right\}, \tag{2}$$

where $\tau$ = proper time measured from exit face of the regenerator; $\alpha$ = an overall normalization constant; $\epsilon(P,\tau)$ = the detection efficiency obtained by Monte Carlo; $\rho(P)$ = the regeneration amplitude; $\eta_{+-}$ = the ratio [amplitude of $K_L \to \pi^+\pi^-$/amplitude of $K_S \to \pi^+\pi^-$]; $\phi_T$ = the total phase angle $\phi_T = \phi_\rho - \phi_{+-}$; $\Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L)$.

The data are organized into an array of $0.5 \times 10^{-10}$ sec proper time bins and 0.5 GeV/$c$ momentum bins and fit to $I_{+-}(P,\tau)$ using the standard fitting program, MINUIT [4]. The fit provides values for $\Gamma_S$, $|\rho/\eta_{+-}|$ and $\phi_T$. $\Gamma_S$ is found to be $[1.122 \pm 0.004] \times 10^{10}\ \mathrm{sec}^{-1}$ while the results for $\phi_T$ and its constituents $\phi_f$, $\phi_{+-}$ appear elsewhere [3,5]. The fitted values for $[f(0) - \bar{f}(0)]/P|\eta_{+-}|$ are tabulated in table 1.

The $\pi^+\pi^-$ data alone are insufficient to determine $|\eta_{+-}|$. However, the $K \to \pi e\nu$ charge asymmetry [3] provides an independent, though less precise, measure of $\rho$. By fitting the charge asymmetry along with the $\pi^+\pi^-$ intensity, we obtain the

Table 1

| Momentum ($GeV/c$) | $[f(0)-\bar{f}(0)]/P\lvert\eta_{+-}\rvert$ (mb) | $[f(0)-\bar{f}(0)]/P$ (mb) [a] |
|---|---|---|
| 4–5 | 2593 ± 29 | 5.91 ± 0.07 |
| 5–6 | 2252 ± 24 | 5.14 ± 0.06 |
| 6–7 | 2000 ± 23 | 4.56 ± 0.05 |
| 7–8 | 1889 ± 29 | 4.31 ± 0.07 |
| 8–9 | 1711 ± 36 | 3.90 ± 0.08 |
| 9–10 | 1726 ± 67 | 3.93 ± 0.15 |

[a] The value of $\lvert\eta_{+-}\rvert$ used here is $2.279 \times 10^{-3}$.

product

$$\frac{1-|x|^2}{|1+x|^2}|\eta_{+-}| = (2.19 \pm 0.05) \times 10^{-3}\,, \tag{3}$$

where $x$ is the ratio of $\Delta S = -\Delta Q$ amplitude to the $\Delta S = +\Delta Q$ amplitude. This can be compared with the average of four recent experiments [6]:

$$|\eta_{+-}| = (2.279 \pm 0.025) \times 10^{-3}\,. \tag{4}$$

Since our measurement [3] is compatible with this average in the limit $x \to 0$, we use this average $|\eta_{+-}|$ to extract $|f(0) - \bar{f}(0)|$ from table 1. The result is plotted in fig. 1. Our results are in good agreement with a previous measurement [7] * in carbon when scaled by the same value for $|\eta_{+-}|$. The dependence of our results on the value of $\Gamma_S$ can be demonstrated in the expression

$$\left|\frac{\rho}{\eta_{+-}}\right| = \left|\frac{\rho}{\eta_{+-}}\right|_{\text{measured}} - 45.3\left[\frac{\Gamma_S - 1.122}{\Gamma_S}\right]. \tag{5}$$

The dependence on $\Delta m$ and $\phi_T$ is minimal.

The amplitude $|f(0) - \bar{f}(0)|$ can be calculated within the framework of the optical model [8–10]. Briefly, if we denote with $a^p(0)$ and $a^n(0)$ the forward scattering amplitudes of $K^0$ on protons and neutrons respectively, the optical theorem and isospin symmetry say that

$$\mathrm{Im}[a^p(0)] = \tfrac{1}{4}k\,\sigma_T(K^0p) = \tfrac{1}{4}k\,\sigma_T(K^+n)\,,$$

$$\mathrm{Im}[a^n(0)] = \tfrac{1}{4}k\,\sigma_T(K^0n) = \tfrac{1}{4}k\,\sigma_T(K^+p)\,, \tag{6}$$

where $\sigma_T$ stands for the total cross section. Similar relationships can be obtained for $\bar{K}$. The real parts $\mathrm{Re}[a^p(0)]$, $\mathrm{Re}[a^n(0)]$ can be obtained from the total cross sections through the use of forward dispersion relations. These nucleon amplitudes are con-

* Bohm et al. [7] obtain $|f - \bar{f}|/P = 6.7 \pm 0.5$ mb at 2.7 $GeV/c$.

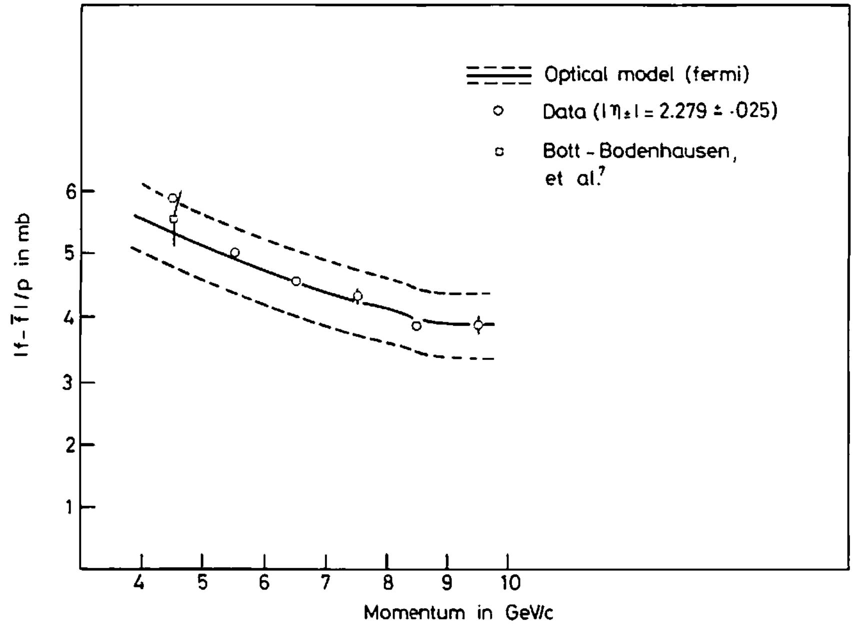


Fig. 1. Comparison of data and optical model predictions.

verted into amplitudes on a nucleus by integrating appropriately over the shape of the nucleus in the plane wave Born approximation.

In our calculations, we have used the real and imaginary parts of nucleon amplitudes calculated by Carter [11]. He determines the imaginary parts from total cross section measurements and real parts using once-subtracted dispersion relations discussed by Lusignoli et al. [12]. For the shape of the nucleus, we have tried both the Fermi and harmonic well distributions.

The Fermi distribution is given by

$$u(r) = \rho_0 \Big/ \left[1 + \exp\left(\frac{r - r_0}{d}\right)\right] , \tag{8}$$

where $d = 0.54 \pm 0.03$ fm and $r_0 = 2.56 \pm 0.16$ fm from electron scattering experiments. The harmonic well distribution was [13]

$$u(r) = \frac{2}{\pi^{3/2}} \frac{1}{a_0^3(2 + 3\alpha)} \left(1 + \frac{\alpha r^2}{a_0^2}\right) e^{-r^2/a_0^2} , \tag{9}$$

where

$\alpha = 1.33$ for carbon .

$a_0 = 1.77$ fm .

The two approaches yield compatible results though the harmonic well values of $|f(0) - \bar{f}(0)|$ are somewhat higher. Uncertainties of the order of 10% have been conservatively estimated from the uncertainties in the nuclear parameters and the real parts of the $K^+$ nuclear forward scattering amplitudes. The agreement with the experimental results is good *, as shown in fig. 1.

The data may also be interpreted in the Regge pole framework. Expanding $f(0)$ and $\bar{f}(0)$ in the usual Regge amplitudes [14,15],

$$f(0) = I\!P + f - \omega\,,$$

$$\bar{f}(0) = I\!P + f + \omega\,, \tag{10}$$

where $I\!P$ is the pomeron contribution and $f$ and $\omega$ are the $I = 0$ meson trajectory contributions, we obtain [15]

$$\frac{f(0) - \bar{f}(0)}{P} = -\beta(0)\left(\frac{1 - e^{-i\pi\alpha_\omega(0)}}{\sin \pi\alpha_\omega(0)}\right) P^{\alpha_\omega(0)-1}$$

$$= \beta'(0)\, e^{-i\pi\alpha_\omega(0)/2}\, P^{\alpha_\omega(0)-1}\,. \tag{11}$$

Here the momentum dependence of $|f(0) - f(0)|/P$ is a simple power law,

$$\frac{|f(0) - f(0)|}{P} = C\, P^{\alpha_\omega(0)-1}\,. \tag{12}$$

Furthermore, from eq. (11) $\phi_f = \arg i[f(0) - f(0)] = -\frac{1}{2}\pi\alpha_\omega(0)$ rad. From ref. [3] we take the result $\phi_f = -40.9 \pm 2.6°$, giving

$$\alpha_\omega(0) = 0.454 \pm 0.029\,. \tag{13}$$

A fit of the data of table 1 to eq. (12) yields $\alpha_\omega(0) = 0.368 \pm 0.025$, ($\chi^2 = 7.9/4$ d.o.f.) somewhat lower than that obtained from the regeneration phase. This is consistent with ref. [16] for regeneration in hydrogen, which found $\alpha_\omega = 0.49 \pm 0.05$ from the regeneration phase, but $\alpha_\omega(0) = 0.3 \pm 0.03$ from the energy dependence. On the other hand, as can be seen in fig. 1, the value of $|f(0) - f(0)|/P$ at 4.5 GeV/$c$ seems higher than the general trend of the rest of the data. This may be too low an energy for the Regge picture to be valid, since resonance effects may persist. If we exclude this point in the fit to eq. (12), we obtain

$$\alpha_\omega(0) = 0.42 \pm 0.04\,, \qquad \text{with } \chi^2 = 5.7/3 \text{ d.o.f.}\,, \tag{14}$$

in good agreement with the regeneration phase result [16]. We believe that the latter result (i.e., for $P > 5$ GeV/$c$) reflects more accurately the value of the $\omega$ intercept both because of its consistency with the phase result and because it is less sensitive to non-Regge contributions. One should note that $\alpha_\omega(0)$ is obtained directly from

* Similar results on copper and uranium have recently been obtained by Dydak et al. [17].

the regeneration phase, while it can be obtained only from knowledge of the momentum dependence of the regeneration amplitude. Small contributions to the regeneration amplitude at any energy can strongly affect the extracted value of $\alpha_\omega(0)$. Combining the results of both methods, we obtain $\alpha_\omega = 0.442 \pm 0.023$, which agrees well with that of ref. [16] and also with the value $\alpha_\omega(0) = 0.433 \pm 0.01$ [15] obtained from measurements of $K^\pm p$ high-energy total cross sections.

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## References

[1] R.H. Good et al., Phys. Rev. 124 (1961) 1223.
[2] W.C. Carithers et al., Phys. Rev. Letters 30 (1973) 1336.
[3] W.C. Carithers et al., Phys. Rev. Letters 34 (1975) 1240.
[4] F. James and M. Roos, CERN Computer 6000 Series Program Library, D506-D516.
[5] W.C. Carithers et al., Phys. Rev. Letters 34 (1975) 1244.
[6] K. Kleinknecht, Proc. 17th Int. Conf. on high-energy physics, London, 1974.
[7] M. Bott-Bodenhausen et al., Phys. Letters 20 (1966) 212;
A. Bohm et al., 1966, unpublished;
K. Kleinknecht, Fortschr. Phys. 21 (1973) 57.
[8] S. Fernback, R. Serber and T.B. Taylor, Phys. Rev. 75 (1949) 1352.
[9] K.M. Watson, Phys. Rev. 89 (1953) 575; 105 (1957) 1388.
[10] J.W. Cronin, R.L. Cool and A. Abashian, Phys. Rev. 107 (1955) 1121.
[11] A.A. Carter, Ph.D. thesis, University of Cambridge (1968).
[12] M. Lusignoli et al., Nuovo Cimento 45A (1966) 792; 49A (1967) 705.
[13] R. Hofstadter, Ann. Rev. Nucl. Sci. 7 (1957) 272.
[14] F. Gilman, Phys. Rev. 171 (1968) 1453;
V. Barger and R.J.N. Phillips, Phys. Letters 33B (1970) 425;
H.S. Goldberg and V.S. Telegdi, Phys. Letters 35B (1971) 327;
K. Kleinknecht, Phys. Letters 30B (1969) 514.
[15] R.E. Hendrick et al., Phys. Rev. D11 (1975) 536.
[16] G.W. Brandenburg et al., Phys. Rev. D9 (1974) 1939.
[17] F. Dydak et al., Nucl. Phys. B102 (1976) 253.