Measurement of the Regeneration Phase in Carbon from 4 to 10 GeV/c*


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A regeneration experiment exploring $K^0_L-K^0_S$ interference in the decay modes $K^0_{L,S,L} \rightarrow \pi^+\pi^-$ and $K^0_{L,S,L} \rightarrow \pi^+\pi^- l^+l^-$ (l=\mu or e) has been performed at the Brookhaven National Laboratory alternating-gradient synchrotron. The regeneration phases in carbon obtained from the time-dependent charge asymmetry of the $K_{L3}$ and $K_{S3}$ modes are in good agreement and yield a combined result $\phi_f = \arg \left( \int f(0) - \int \bar{f}(0) \right) = -40.9^\circ \pm 2.6^\circ$ at the average $K^0$ momentum of 7.5 GeV/c.

Since the discovery of CP nonconservation, the phenomenon of coherent $K^0$ regeneration from an initially pure $K^0_L$ beam has been exploited in a number of experiments designed to measure the phase $\phi_+$. of the CP-nonconservation parameter: $\eta_+ = \langle \pi^+\pi^- | \mathcal{T} | K^0_L \rangle / \langle \pi^+\pi^- | \mathcal{T} | K^0_S \rangle = \eta_+ | \exp(i\phi_+) | . (1)$

The principal limitation in this type of experiment arises from the uncertainty of the regeneration phase, $\phi^0$, which enters directly in any interference of the $K^0_L$ and regenerated $K^0_S$ amplitudes.

After traversing a block of matter (the regenerator), a pure $|K^0_L\rangle$ beam is transformed into a coherent mixture $\psi = a|K^0_L\rangle + b|K^0_S\rangle$. The regeneration amplitude, defined at the exit face of the regenerator for the undeflected beam, is

$$\rho = b/a = |\rho| \exp(i\phi_\rho) = \frac{i\pi NL[f(0) - \bar{f}(0)]}{P_K} \left[ 1 - \exp \left( \frac{i\Delta m - \Gamma_\rho/2}{\Delta m} L M_\rho / P_K \right) \right]$$

$$- \left( \frac{i\Delta m - \Gamma_\rho/2}{\Delta m} L M_\rho / P_K \right),$$

where $N$ is the atomic density, $L$ is the length of the regenerator, $P_K$ is the $K$ momentum, and $f(\bar{f})$ is the $K^0$ ($\bar{K}^0$) nucleus forward scattering amplitude. In the determination of $\phi_\rho$, the poorest known part is $\phi_\rho = \arg[f(0) - \bar{f}(0)].$

Several methods have been employed for the determination of $\phi_\rho$. The method followed here utilizes the time-dependent charge asymmetry in the decay modes $K^0_{L,S} \rightarrow \pi^+\pi^- l^+l^-$, where $l$ is either a muon or electron. The charge asymmetry after a regenerator is defined by $\delta(\tau) = \frac{\Gamma_\rho(\tau) - \Gamma_{\bar{\rho}}(\tau)}{\Gamma_\rho(\tau) + \Gamma_{\bar{\rho}}(\tau)}$, where $\Gamma_\rho$ ($\Gamma_{\bar{\rho}}$) refers to positive (negative) leptonic decay rate, and is given to sufficient accuracy for the present discussion by

$$\delta(\tau) = 2\chi [\rho \exp(-(\Gamma_\rho + \Gamma_{\bar{\rho}})\tau/2) \left\{ \cos(\Delta m\tau + \phi_\rho) + \alpha \cos(\Delta m\tau + \phi - \phi_\rho) \right\} + \delta_\rho \right].$$

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Here \( \chi = \frac{1 - |x|^3}{1 - |x|^2} \), where \( x \) is the ratio of possible \( \Delta S = - \Delta Q \) amplitudes to \( \Delta S = \Delta Q \) amplitudes, \( \tau \) is proper time measured from the exit face of the regenerator, and \( \delta_L \) is the observed CP-nonconserving \( K_\ell \) charge asymmetry. The first term enclosed by the curly brackets of Eq. (3) arises from the coherently produced (transmission regeneration) events; \( \alpha \) and \( \varphi' \) are introduced to account for incoherently produced events caused by diffractive and/or inelastic scattering within the regenerator. At short proper time, the observed asymmetry can be of the order of 0.1, whereas the asymptotic value given by \( \delta_L \) is known to be \( \sim (3.4 \pm 0.2) \times 10^{-3} \).

The apparatus (Fig. 1) is essentially the same as that used to search for \( K_\ell - \mu \mu \), except for the replacement of the vacuum decay region by a regenerator assembly and helium decay region. The bulk of the data was taken with a carbon regenerator 81.28 cm long, density 1.72 g/cm\(^3\), and with chemical impurities measured to be less than 0.1%. The veto counter, in contact with the exit face, was 0.32 cm thick and dc coupled to dead-time-free electronics. The event trigger required the traversal of the spectrometer by exactly two tracks, and that no veto count was present. The magnetic field was reversed regularly to allow the cancelation of possible apparatus asymmetry.

The analysis of the semileptonic decays is substantially more complicated than that of the \( \pi^+ \pi^- \) mode because of the missing kinematic information for the neutrino. It is impossible to separate event by event the coherently produced events from the incoherently produced events, and the calculation of the \( K \) momentum (and hence proper time) contains an inherent twofold ambiguity. Space limitations preclude a complete discussion of the analysis here, and an appropriate presentation will be published elsewhere.

Constraints on event reconstruction were described by the following quantities: (1) the observed "kink" angle of each trajectory in a plane parallel to the magnetic field; (2) the closest distance of approach (CDA) of the trajectories in the decay region; (3) the variable \( \Delta \nu = |P_{\nu l}, - P_{\ell} \), i.e., the magnitude of the neutrino momentum as determined from the invariant mass of the pion-lepton system minus the observed transverse momentum of the pion-lepton system; (4) the visible longitudinal momentum \( P_\ell = (P_\nu + P_\ell), \) and (5) the invariant mass computed under various decay-mode hypotheses. For each event, a \( \chi^2 \) sum for the two trajectory kinks and CDA was computed.

The following requirements were applied to the \( K_{\mu 3} \) decay candidates: (1) \( \chi^2 < 10 \), designed to reject erroneously matched track segments, \( \pi^- \mu^+ \) decays within the spectrometer, and other backgrounds; (2) \( M_{\pi \mu} < 460 \text{ MeV}/c^2 \), intended to reject feedthrough of the relatively copious \( K_s \) \( \pi^+ \pi^- \) decays; (3) \( M_{\pi \pi} \) outside the interval 1105–
1125 MeV/c^2, designed to reject feedthrough of 
\(\Lambda \to p n\) decays; (4) \(-2 < \Delta \nu < 118\) MeV/c, designed
to discriminate against the incoherent events;
(5) \(P_{p'} \ll 10\) GeV/c; (6) \(1 < P_{p} < 8\) GeV/c; (7) \(P_{\mu}
> 2\) GeV/c; (8) various fiducial-volume, spectrometer,
and muon-hodoscope boundary cuts
to ensure proper registration of the event; (9) in-
bending trajectories, i.e., both trajectories are
bent towards the beam center line; and (10) ex-
actly one muon track, defined by appropriate
counts in both horizontal and vertical hodoscope
arrays along the extrapolated trajectory. The
number of events which satisfy the \(K_{\mu3}\) criteria
is \(3.5 \times 10^6\), with an estimated background less
than \(1\%\).

The \(K_{e3}\) decays are less prone to background
contamination and have been subjected to less
restrictive requirements: (1) \(\chi^2 \ll 16\); (2) \(\Delta \nu
> -2\) MeV/c; (3) \(P_{p'} < 11\) GeV/c; (4) \(1 < P_{\mu} < 7\)
GeV/c; (5) \(1 < P_{\mu} < 8\) GeV/c; (6) appropriate fi-
ducial, spectrometer, and Cherenkov acceptance
cuts; and (7) inbending trajectories. The num-
ber of events passing the \(K_{e3}\) criteria is \(7 \times 10^6\)
with negligible background.

The twofold ambiguity in the proper-time com-
putation has been dealt with by defining an appar-
ent proper time \(\tau' = M_P (z - z_{veto}) / P_{p'}\), where \(z\)
is the location of the decay vertex given by the
CDA calculation. The relationship of \(\tau\) and \(\tau'\)
was then obtained by use of a Monte Carlo-de-
derived transformation matrix \(A\): \(\delta (\tau') = \sum_x A(\tau, \tau') \times \delta (\tau)\). The requisite accuracy of the Monte Carlo
simulation was carefully checked by high-statistics
comparisons of a large number of variable
distributions for both \(K^{0} \to \pi^+ \pi^-\) and \(K^{0} \to \pi^0 \mu^+ \nu\)
modes. As the regeneration phase and amplitude
are both expected to vary slowly with momentum,
the data have been analyzed separately in 1 GeV/c
\(P_{p'}\) intervals. For clarity of presentation here,
however, the data have been summed over mo-
momentum and are shown in Fig. 2.

The understanding and treatment of the inelas-
tically and diffractionally produced events have
been facilitated by a comparison of the easily
separated coherent and incoherent \(K^{0} \to \pi^+ \pi^-\)
events. Unlike previous work, the present trig-
ger accepts the latter with relatively high effi-
ciency. The detailed analysis, 7 lengthy but
straightforward, shows that with judiciously cho-

\[\varphi' = \left(-41.6^\circ \pm 2.6^\circ\right) - 65^\circ \left(\Delta m - 0.540\right) / \Delta m \right) - 30^\circ \left(\Gamma_s - 1.124 / 1.124\right).\]

FIG. 2. (a) \(K_{e3}\) charge asymmetry versus apparent
proper time, summed over momentum. (b) \(K_{\mu3}\) charge
asymmetry.

sen cuts the impact of the incoherent events on
the measured charge asymmetry can be made
small [see Eq. (3)]:

\[K_{e3}: \quad \varphi' = 9^\circ \pm 3^\circ\]

\[K_{\mu3}: \quad \varphi' = 11^\circ \pm 3^\circ.\]

By use of the optimization program MINUIT 8 the
data were simultaneously fitted for \(\chi [\beta]\) and \(\varphi'\),
with \(\Delta m\) fixed at 0.540 \times 10^{10} \text{sec}^{-1},\ \Gamma_s\ fixed at
1.124 \times 10^{10} \text{sec}^{-1},\ \text{and } \delta / \beta\ fixed at the observed
\(K_L^0\) asymmetry.9 The \(\chi^2\) sum for the \(K_{e3}\) data is
172 for 194 degrees of freedom; for the \(K_{\mu3}\) data,
\(\chi^2\) is 87 for 101 degrees of freedom. The results
are given in Table I and shown in Fig. 3 of the
following Letter.

The momentum dependence of \(\varphi'\) is only poorly
determined from the leptonic data. At the mean
momentum of 7.5 GeV/c, the results are

\[K_{e3}: \quad \varphi' = -41.2^\circ \pm 3.2^\circ,\]

\[K_{\mu3}: \quad \varphi' = -42.5^\circ \pm 4.6^\circ,\]

in good agreement with each other.

The combined result, including the observed
dependence on \(\Delta m\) and \(\Gamma_s\), is

\[\varphi' = \left(-41.6^\circ \pm 2.6^\circ\right) - 65^\circ \left(\Delta m - 0.540\right) / \Delta m \right) - 30^\circ \left(\Gamma_s - 1.124 / 1.124\right).\]
Subsequent to the completion of the analysis, new results for $\Delta m$ have become available which significantly affect the average $\Delta m$ value and uncertainty. For reasons discussed in the following Letter, we prefer to employ the value $\Delta m/h = (0.5348 \pm 0.0021) \times 10^{10}$ sec$^{-1}$, leading to the result $\varphi_f = -40.9^\circ \pm 2.6^\circ$.

This result may be compared with an optical-model calculation using input values of $\text{Re} f(0)$ and $\text{Re} f(0)$ obtained by dispersion-relation calculations. The results of Lusignoli et al.$^{10}$ yield $\varphi_f = -41^\circ \pm 7^\circ$ whereas those of Carter$^{11}$ yield $\varphi_f = 50^\circ \pm 7^\circ$. The errors quoted are estimates of the input errors and do not reflect possible inadequacies of the optical model in this context.

Nevertheless, the experimental results for $\varphi_f$, given in Table I, may be directly utilized in the analysis of interference in $K_{e3} \rightarrow \pi^+\pi^-$ decays, the subject of the following Letter.

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9The value of $\Delta m$ as given by V. Chaloupka, Phys. Lett. 50B, 1 (1974); the value of $\Gamma_S$ as given in the following Letter; the value of $\delta_\alpha$ as given in Ref. 5.
