(1964)].
[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).
[2]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974).
[3]G. S. Abrams *et al.*, Phys. Rev. Lett. 33, 1453 (1974).
[4]A. De Rújula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1975); T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, Phys. Rev. Lett. 34, 365 (1975); S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38, 236 (1975); E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T. M. Yan, Phys. Rev. Lett. 34, 369 (1975); T. Goto and V. S. Mathur, to be published.
[5]G. Snow, Nucl. Phys. B55, 445 (1973); M. K. Gaillard, B. W. Lee, and J. L. Rosner, FNAL Report No. Lab-Pub-74/86-THY, 1974 (Rev. Mod. Phys., to be published).
[6]T. Ferbel, private communication.
[7]P. L. Jain and B. Girard, to be published.
[8]E. Wang and P. L. Jain, Nuovo Cimento 56A, 922 (1968).

Measurement of the Regeneration Phase in Carbon from 4 to 10 GeV/c*

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A regeneration experiment exploring K_S-K_L interference in the decay modes $K_{S,L} \to \pi^+\pi^-$ and $K_{S,L} \to \pi^\pm l^\mp \nu$ ($l=\mu$ or e) has been performed at the Brookhaven National Laboratory alternating-gradient synchrotron. The regeneration phases in carbon obtained from the time-dependent charge asymmetry of the K_{e3} and $K_{\mu3}$ modes are in good agreement and yield a combined result $\varphi_f \equiv \arg i[f(0)-\bar{f}(0)] = -40.9° \pm 2.6°$ at the average K^0 momentum of 7.5 GeV/c.

Since the discovery of CP nonconservation,[1] the phenomenon of coherent K_S regeneration from an initially pure K_L beam has been exploited in a number of experiments[2] designed to measure the phase φ_{+-} of the CP-nonconservation parameter,[3]

$$\eta_{+-} \equiv \langle \pi^+\pi^- | T | K_L \rangle / \langle \pi^+\pi^- | T | K_S \rangle = |\eta_{+-}| \exp(i\varphi_{+-}). \tag{1}$$

The principal limitation in this type of experiment arises from the uncertainty of the regeneration phase, φ_ρ, which enters directly in any interference of the K_L and regenerated K_S amplitudes.

After traversing a block of matter (the regenerator), a pure $|K_L\rangle$ beam is transformed into a coherent mixture $\psi = a|K_L\rangle + b|K_S\rangle$. The regeneration amplitude, defined at the exit face of the regenerator for the undeflected beam, is

$$\rho \equiv b/a = |\rho| \exp(i\varphi_\rho) = \frac{i\pi NL[f(0)-\bar{f}(0)]}{P_K} \, \frac{1-\exp[(i\Delta m - \Gamma_s/2)LM_K/P_K]}{-(i\Delta m - \Gamma_s/2)LM_K/P_K}, \tag{2}$$

where N is the atomic density, L is the length of the regenerator, P_K is the K momentum, and f ($\bar{f}$) is the K^0- ($\bar{K}^0$-) nucleus forward scattering amplitude. In the determination of φ_ρ, the poorest known part is $\varphi_f \equiv \arg i[f(0)-\bar{f}(0)]$.

Several methods have been employed for the determination of φ_f.[4] The method followed here utilizes the time-dependent charge asymmetry in the decay modes $K_{L,S} \to \pi^\pm l^\mp \nu$, where l is either a muon or electron. The charge asymmetry after a regenerator is defined by $\delta(\tau) \equiv [\Gamma_+(\tau) - \Gamma_-(\tau)]/[\Gamma_+(\tau) + \Gamma_-(\tau)]$, where Γ_+ (Γ_-) refers to positive (negative) leptonic decay rate, and is given to sufficient accuracy for the present discussion by

$$\delta(\tau) = 2\chi|\rho| \exp[-(\Gamma_S + \Gamma_L)\tau/2]\{\cos(\Delta m\tau + \varphi_\rho) + \alpha\cos(\Delta m\tau + \varphi - \varphi')\} + \delta_L. \tag{3}$$

Here $\chi = [1-|x|^2]/|1-x|^2$, where x is the ratio of possible $\Delta S = -\Delta Q$ amplitudes to $\Delta S = +\Delta Q$ amplitudes, τ is proper time measured from the exit face of the regenerator, and δ_L is the observed CP-nonconserving $K_L{}^0$ charge asymmetry. The first term enclosed by the curly brackets of Eq. (3) arises from the coherently produced (transmission regeneration) events; α and φ' are introduced to account for incoherently produced events caused by diffractive and/or inelastic scattering within the regenerator. At short proper time, the observed asymmetry can be of the order of 0.1, whereas the asymptotic value given by δ_L is known[5] to be $\sim(3.4\pm0.2)\times10^{-3}$.

The apparatus (Fig. 1) is essentially the same as that used to search for $K_L \rightarrow \mu^+\mu^-$,[6] except for the replacement of the vacuum decay region by a regenerator assembly and helium decay region. The bulk of the data was taken with a carbon regenerator 81.28 cm long, density 1.72 g/cm^3, and with chemical impurities measured to be less than 0.1%. The veto counter, in contact with the exit face, was 0.32 cm thick and dc coupled to dead-time-free electronics. The event trigger required the traversal of the spectrometer by exactly two tracks, and that no veto count was present. The magnetic field was reversed regularly to allow the cancelation of possible apparatus asymmetry.

The analysis of the semileptonic decays is substantially more complicated than that of the $\pi^+\pi^-$ mode because of the missing kinematic information for the neutrino. It is impossible to separate event by event the coherently produced events from the incoherently produced events, and the calculation of the K momentum (and hence proper time) contains an inherent twofold ambiguity. Space limitations preclude a complete discussion of the analysis here, and an appropriate presentation will be published elsewhere.

Constraints on event reconstruction were described by the following quantities: (1) the observed "kink" angle of each trajectory in a plane parallel to the magnetic field; (2) the closest distance of approach (CDA) of the trajectories in the decay region; (3) the variable $\Delta\nu \equiv |P_\nu|_{\mathrm{c.m.}} - P_\perp$, i.e., the magnitude of the neutrino momentum as determined from the invariant mass of the pion-lepton system minus the observed transverse momentum of the pion-lepton system; (4) the visible longitudinal momentum $P_K' \equiv (P_\pi + P_l)_z$; and (5) the invariant mass computed under various decay-mode hypotheses. For each event, a χ^2 sum for the two trajectory kinks and CDA was computed.

The following requirements were applied to the $K_{\mu3}$ decay candidates: (1) $\chi^2 \leq 10$, designed to reject erroneously matched track segments, $\pi \rightarrow \mu\nu$ decays within the spectrometer, and other backgrounds; (2) $M_{\pi\pi} < 460$ MeV/c^2, intended to reject feedthrough of the relatively copious $K_S \rightarrow \pi^+\pi^-$ decays; (3) $M_{p\pi}$ outside the interval 1105–

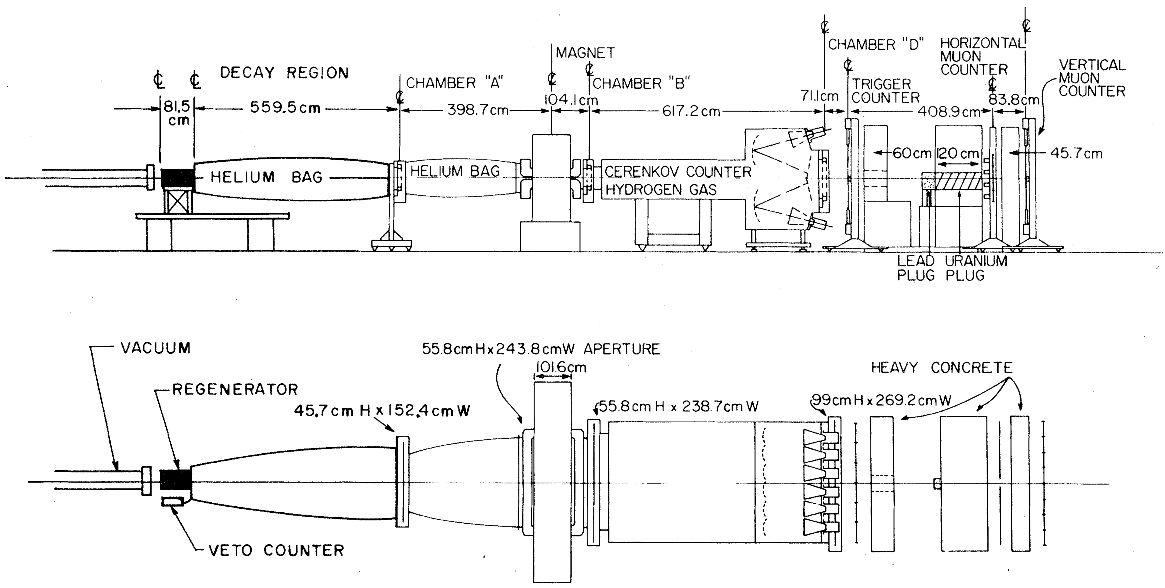

FIG. 1. Views of the apparatus. Chambers A, B, and D are multiwire proportional chambers. After passing through a hole in the wall directly behind the trigger counters, the neutral beam was absorbed in a lead and uranium insert.

1125 MeV/c^2, designed to reject feedthrough of $\Lambda \to p\pi$ decays; (4) $-2 < \Delta\nu < 118$ MeV/c, designed to discriminate against the incoherent events; (5) $P_K' \leq 10$ GeV/c; (6) $1 \leq P_\pi \leq 8$ GeV/c; (7) $P_\mu > 2$ GeV/c; (8) various fiducial-volume, spectrometer, and muon-hodoscope boundary cuts to ensure proper registration of the event; (9) inbending trajectories, i.e., both trajectories are bent towards the beam center line; and (10) exactly one muon track, defined by appropriate counts in both horizontal and vertical hodoscope arrays along the extrapolated trajectory. The number of events which satisfy the $K_{\mu 3}$ criteria is 3.5×10^6, with an estimated background less than 1%.

The K_{e3} decays are less prone to background contamination and have been subjected to less restrictive requirements: (1) $\chi^2 \leq 16$; (2) $\Delta\nu > -2$ MeV/c; (3) $P_K' < 11$ GeV/c; (4) $1 < P_e < 7$ GeV/c; (5) $1 < P_\pi < 8$ GeV/c; (6) appropriate fiducial, spectrometer, and Cherenkov acceptance cuts; and (7) inbending trajectories. The number of events passing the K_{e3} criteria is 7×10^6 with negligible background.

The twofold ambiguity in the proper-time computation has been dealt with by defining an apparent proper time $\tau' = M_K(z - z_{\text{veto}})/P_K'$, where z is the location of the decay vertex given by the CDA calculation. The relationship of τ and τ' was then obtained by use of a Monte Carlo–derived transformation matrix A: $\delta(\tau') = \sum_\tau A(\tau,\tau') \times\delta(\tau)$. The requisite accuracy of the Monte Carlo simulation was carefully checked by high-statistics comparisons of a large number of variable distributions for both $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi l \nu$ modes. As the regeneration phase and amplitude are both expected to vary slowly with momentum, the data have been analyzed separately in 1-GeV/c P_K' intervals. For clarity of presentation here, however, the data have been summed over momentum and are shown in Fig. 2.

The understanding and treatment of the inelastically and diffractively produced events have been facilitated by a comparison of the easily separated coherent and incoherent $K^0 \to \pi^+\pi^-$ events. Unlike previous work, the present trigger accepts the latter with relatively high efficiency. The detailed analysis,[7] lengthy but straightforward, shows that with judiciously chosen cuts the impact of the incoherent events on the measured charge asymmetry can be made small [see Eq. (3)]:

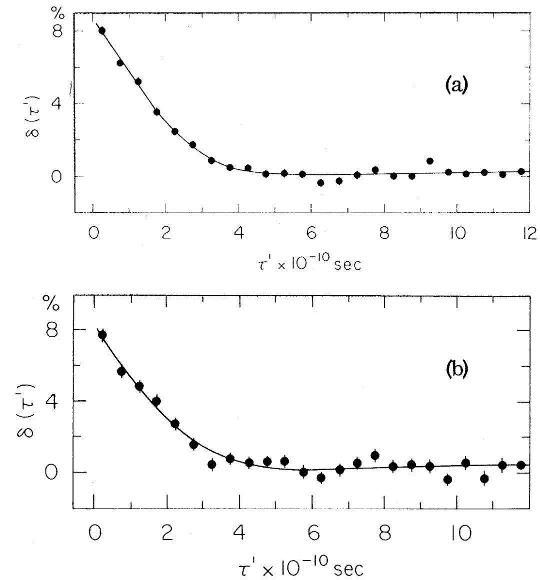

FIG. 2. (a) K_{e3} charge asymmetry versus apparent proper time, summed over momentum. (b) $K_{\mu 3}$ charge asymmetry.

$$\begin{aligned} &K_{e3}\text{: } \alpha = 0.156 \pm 0.014, \quad \varphi' = 10° \pm 3° \\ &K_{\mu 3}\text{: } \alpha = 0.14 \pm 0.02, \quad \varphi' = 10° \pm 3°. \end{aligned} \tag{4}$$

By use of the optimization program MINUIT,[8] the data were simultaneously fitted for $\chi|\rho|$ and φ_ρ, with Δm fixed at 0.540×10^{10} sec^{-1}, Γ_s fixed at 1.124×10^{10} sec^{-1}, and δ_L fixed at the observed $K_L{}^0$ asymmetry.[9] The χ^2 sum for the K_{e3} data is 172 for 194 degrees of freedom; for the $K_{\mu 3}$ data, χ^2 is 87 for 101 degrees of freedom. The results are given in Table I and shown in Fig. 3 of the following Letter.

The momentum dependence of φ_f is only poorly determined from the leptonic data. At the mean momentum of 7.5 GeV/c, the results are

$$\begin{aligned} &K_{e3}\text{: } \varphi_f = -41.2° \pm 3.2°, \\ &K_{\mu 3}\text{: } \varphi_f = -42.5° \pm 4.6°, \end{aligned} \tag{5}$$

in good agreement with each other.

The combined result, including the observed dependence on Δm and Γ_S, is

$$\varphi_f = (-41.6° \pm 2.6°) - 65°\left(\frac{\Delta m - 0.540}{\Delta m}\right) - 30°\left(\frac{\Gamma_s - 1.124}{1.124}\right). \tag{6}$$

TABLE I. Results for the regeneration phase referred to values $\Delta m = 0.540$ and $\Gamma_S = 1.1124$. $\langle P_K \rangle$ is the true mean K^0 momentum for the indicated P_K' interval. Momentum units are GeV/c; φ units are rad.

	K_{e3}		$K_{\mu 3}$	
P_K'	$\langle P_K \rangle$	φ_f	$\langle P_K \rangle$	φ_f
3–4	4.5	-0.64 ± 0.20	5.6	-0.66 ± 0.17
4–5	5.6	-0.86 ± 0.13	6.7	-1.00 ± 0.18
5–6	6.6	-0.82 ± 0.12	7.7	-0.64 ± 0.16
6–7	7.6	-0.74 ± 0.12	8.7	-0.63 ± 0.18
7–8	8.5	-0.56 ± 0.17	10.0	-0.79 ± 0.19
8–9	9.5	-0.88 ± 0.18		
9–10	10.4	-0.26 ± 0.20		
10–11	11.2	-0.66 ± 0.42		

Subsequent to the completion of the analysis, new results for Δm have become available which significantly affect the average Δm value and uncertainty. For reasons discussed in the following Letter, we prefer to employ the value $\Delta m/\hbar = (0.5348 \pm 0.0021) \times 10^{10}$ sec^{-1}, leading to the result $\varphi_f = -40.9° \pm 2.6°$.

This result may be compared with an optical-model calculation using input values of Re$f(0)$ and Re$\bar{f}(0)$ obtained by dispersion-relation calculations. The results of Lusignoli *et al.*[10] yield $\varphi_f = -41° \pm 7°$ whereas those of Carter[11] yield $\varphi_f = 50° \pm 7°$. The errors quoted are estimates of the input errors and do not reflect possible inadequacies of the optical model in this context.

Nevertheless, the experimental results for φ_f given in Table I can be directly utilized in the analysis of interference in $K_{S,L} \to \pi^+\pi^-$ decays, the subject of the following Letter.

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[1]J. H. Christenson *et al.*, Phys. Rev. Lett. 13, 138 (1964).

[2]S. Bennett *et al.*, Phys. Lett. 27B, 248 (1968), and 29B, 317 (1969); H. Faissner *et al.*, Phys. Lett. 30B, 204 (1969); M. Ya. Balats *et al.*, Yad. Fiz. 13, 93 (1971) [Sov. J. Nucl. Phys. 13, 53 (1971)]; R. K. Carnegie *et al.*, Phys. Rev. D 6, 2335 (1972).

[3]T. T. Wu and C. N. Yang, Phys. Rev. Lett. 13, 380 (1964).

[4]M. Bott-Bodenhausen *et al.*, Phys. Lett. 24B, 438 (1967); C. Rubbia and J. Steinberger, Phys. Lett. 23, 167 (1966).

[5]G. Geweniger *et al.*, Phys. Lett. 48B, 483 (1974).

[6]W. C. Carithers *et al.*, Phys. Rev. Lett. 30, 1336 (1973).

[7]The analysis can be found in T. P. Pun, Ph.D. thesis, Nevis Laboratories, Columbia University, Report No. 204 (unpublished); H. Sticker, Ph.D. thesis, Nevis Laboratories, Columbia University, Report No. 205 (unpublished).

[8]F. James and M. Roos, CERN Computer 6000 Series Program Library, D506–D516 (unpublished).

[9]The value of Δm as given by V. Chaloupka, Phys. Lett. 50B, 1 (1974); the value of Γ_S as given in the following Letter; the value of δ_L as given in Ref. 5.

[10]M. Lusignoli *et al.*, Nuovo Cimento 45A, 792 (1966), and 49A, 705 (1967).

[11]A. A. Carter, Ph.D. thesis, University of Cambridge, 1968 (unpublished).